

The ‘Tip’ of the Red Giant Branch as a distance indicator: results from evolutionary models

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ABSTRACT

New theoretical evaluations of the Red Giant Branch Tip (TRGB) luminosity, by adopting the most updated physical inputs in computing canonical stellar models, are presented. Theoretical relations for the run of the TRGB bolometric and I magnitude with the metallicity are provided together with a comparison of the distance scale based on these relations and the RR Lyrae distance scale presented in Cassisi & Salaris (1996) and the Cepheid distance scale adopted by Lee, Freedman & Madore (1993). The result of this comparison - performed by adopting a sample of galactic globular clusters and a sample of resolved galaxies - discloses a satisfactory agreement between theory and observations at the level of 0.1 mag. This occurrence can be regarded as an evidence for the consistency between theoretical Red Giant, Horizontal Branch stellar models and independent Cepheid observations, and allows to safely use the provided TRGB luminosity relations as an alternative primary distance indicator for galaxies in which the stellar component has been resolved.

Key words: stars: evolution – stars: interiors – globular clusters: general – galaxies: distances and redshifts

1 INTRODUCTION

The Cepheid period-luminosity (P-L) relation is the basis for the calibration of a wide range of secondary distance indicators applicable at larger distances than the Cepheids themselves. However, Cepheid observations are restricted only to Population I systems and to late-type galaxies, while an excellent alternative primary distance indicator is the tip of the Red Giant Branch (TRGB); the use of this indicator results particularly attractive since it is applicable to all morphological types of galaxies as long as an old stellar population is present. After the pioneering studies by Baade (1944) and Sandage (1971) suggesting the potential usefulness of the TRGB as a distance indicator, the TRGB method for estimating the distances to several nearby galaxies was used by many authors (see e.g. Lee, Freedman & Madore 1993, hereinafter LFM93; Sakai, Madore & Freedman 1996, Soria et al. 1996, Elson 1996).

The TRGB method has an underlying physical basis: the tip of the Red Giant Branch (RGB) marks the helium ignition in the degenerate He core of low mass stars, and its luminosity depends on the He core mass, that is remarkably constant for ages larger than a few Gyr. According to models found in the literature (see e.g. Straniero & Chieffi 1991, Castellani, Chieffi & Straniero 1992 and references therein) the bolometric luminosity of the TRGB (at a fixed metallicity Z between 0.0001 and 0.02) varies by at most 0.05 mag for ages ranging from 10 up to 20 Gyr, and by ≈ 0.1 mag if the range is stretched down to ≈ 2 Gyr (the exact value depending on the metallicity). LFM93 have shown

that taking into account the Yale isochrones (Green, Demarque & King 1987) and the photometric data provided by Da Costa & Armandroff (1990, hereinafter DA90) the theoretical and observational I magnitude of TRGB stars in globular clusters (GCs) is constant within 0.1 mag for $-2.2 < [Fe/H] < -0.7$, when the corresponding V magnitude varies by 1.3 mag over the same metallicity range (see their fig. 1), suggesting therefore the use of the observed I magnitude of TRGB stars as a distance indicator.

In particular, LFM93 have provided a calibration of this method in a large range of metallicity, and in a subsequent paper Madore & Freedman (1995) undertook a number of computer simulations and concluded that the TRGB method can be successfully used to determine distances accurate to 0.2 mag for galaxies out to 3 Mpc using ground based telescopes, and out to a factor of four further in distance using the Hubble Space Telescope.

In the present paper, we want to investigate the possibility to adopt the TRGB method for determining distances of nearby galaxies, using results from stellar models computed by adopting the most updated physical inputs. More in details, this paper is the third one of a series (see Salaris & Cassisi 1996 and Cassisi & Salaris 1997, hereinafter Paper I and II respectively) devoted to the comparison of standard updated RGB stellar models with observational data. In Paper I we have discussed the calibration of the effective temperature of RGB stellar models, while in Paper II it is shown that updated RGB models reproduce well the observed V magnitude of GC RGB luminosity function bumps (V_{bump}), which can be used as a diagnostic of the Hydro-

gen stratification in the stellar interior. In paper II we also presented new Zero Age Horizontal Branch (ZAHB) models, computed with the same updated input physics as the RGB ones; the distance scale obtained from these models has been adopted for comparing theoretical and observed values of V_{bump} .

In this paper we will complete the analysis of our updated RGB evolutionary models by comparing the observational determinations of the TRGB luminosities of a sample of GCs and galaxies with theory, adopting the RR Lyrae distance scale given in Paper II and the Cepheid distance scale used by LFM93. In this way we will assess in an independent way the reliability of our theoretical evolutionary models. We provide new updated prescriptions for the bolometric and I (Cousins) magnitude of the RGB tip and for the (V-I) color of observed RGB as a function of the metallicity, based on our theoretical models and on recent spectroscopical determinations of the chemical composition in GC stars, to be applied when using the TRGB method as a distance indicator.

The theoretical RGB models are presented in section 2, while in section 3 the comparison with observational data of a sample of GCs and resolved galaxies is presented. Summary and conclusions follow in section 4.

2 RGB THEORETICAL MODELS

The models used in this paper have been already presented in Paper I and II. To summarize, we have computed canonical evolutionary models of stars with masses of $0.75M_{\odot}$, $0.80M_{\odot}$ and $0.90M_{\odot}$ and for metallicities $Z=0.0001 - 0.0003 - 0.0006 - 0.001 - 0.003 - 0.006$. As for the original Helium abundance (Y), we have adopted $Y=0.23$ at all metallicities, according to the results by Buzzoni et al. (1983), which find an almost constant He abundance in a sample of GCs spanning approximately the same range of metallicity, and to the most recent evaluations for the primordial He abundance (Olive, Skillman & Steigman 1996).

All the models have been computed adopting the FRANEC evolutionary code (see Chieffi & Straniero 1989). The OPAL opacity tables (Rogers & Iglesias 1992, Iglesias, Rogers & Wilson 1992) for $T > 10000K$ and the Alexander & Ferguson (1994) opacities for lower temperatures have been used. Both high and low temperature opacity tables have been computed adopting the solar heavy elements distribution by Grevesse (1991). The electronic conduction is treated according to Itoh et al. (1983). The equation of state (EOS) by Straniero (1988) has been used, supplemented by a Saha EOS at lower temperatures, as described by Chieffi & Straniero (1989).

By interpolating among the various computed stellar models at each metallicity, we have obtained the value of the TRGB luminosity corresponding to an age $t=15$ Gyr. Concerning this choice for the average age the interested reader is referred to Paper II and references therein. However it is worth noting that for old stellar systems as the ones we are dealing with, the TRGB luminosity varies only by $\Delta M_{bol} \leq 0.05$ mag for ages ranging between 10 and 20 Gyr. So we can safely state that the TRGB luminosity is approximately constant in this age interval. For obtaining a variation of the TRGB luminosity of about $\Delta M_{bol} \approx 0.10$

Table 1. Luminosity and bolometric magnitude of the TRGB for the adopted metallicities as obtained by updated stellar models.

Z	$[M/H]$	$\log(L/L_{\odot})^{tip}$	M_{bol}^{tip}
0.0001	-2.35	3.296	-3.490
0.0003	-1.87	3.335	-3.587
0.0006	-1.57	3.361	-3.653
0.001	-1.35	3.378	-3.695
0.003	-0.87	3.416	-3.790
0.006	-0.57	3.438	-3.845

mag, the age of the stellar population has to be largely decreased. By computing additional evolutionary models with higher masses at metallicities $Z=0.0001$ and $Z=0.006$ we find, for instance, that at $Z=0.0001$ a reduction of about $\Delta M_{bol} \approx 0.10$ mag in the TRGB luminosity is obtained for an age around 2.8Gyr, while at $Z=0.006$ the same variation can be found decreasing the age till about 2.2Gyr. This occurrence is related to the behavior of the TRGB luminosity through the Red Giant Phase Transition (Sweigart, Greggio & Renzini 1989,1990), i.e. the luminosity of TRGB is almost constant until the evolving mass on the RGB is lower than the critical mass M_{HeF} which marks the transition between full degenerate stellar structures which ignite the He burning by a recurrent series of flashes inside the He core and the ones which ignite quietly the He burning inside a mild degenerate core.

In Table 1, the TRGB luminosity and its bolometric magnitude (obtained adopting for the Sun $M_{bol} = 4.75$ mag) are reported for each adopted metallicity by assuming, as discussed above, an age equal to 15Gyr. By considering $[M/H] = \log(M/H)_{star} - \log(M/H)_{\odot} \approx \log(Z) + 1.65$, from the data displayed in Table 1 the following relation is obtained:

$$M_{bol}^{tip} = -3.949 - 0.178 \cdot [M/H] + 0.008 \cdot [M/H]^2 \quad (1)$$

for $-2.35 \leq [M/H] \leq -0.57$, with $\sigma = 0.002$.

The enhancement of the α elements observed in galactic field halo and GCs stars (see, e.g. the review by Wheeler, Sneden & Truran 1989) is automatically taken into account by equation 1 when considering the global metallicity $[M/H]$. In fact, as already demonstrated by Salaris, Chieffi & Straniero (1993), Paper I, Salaris, Degl'Innocenti & Weiss (1997), α -enhanced theoretical models are well reproduced by solar scaled ones with the same global metallicity. For fixed values of $[\alpha/Fe] \geq 0$ and $[Fe/H]$ the global metallicity $[M/H]$ is given by (see Salaris et al. 1993):

$$[M/H] \approx [Fe/H] + \log(0.638 \cdot f + 0.362) \quad (2)$$

where $\log(f) = [\alpha/Fe]$.

Equation 1 depends on the adopted initial Helium content since a variation of Y at a fixed metallicity changes the TRGB luminosity because the change of the He core mass at the He flash. As it is well known, the knowledge of the "correct" He enrichment ratio is a longstanding problem, and an exhaustive discussion about this point is beyond the scope of the present work. However, we want to recall that regardless of the adopted $\Delta Y/\Delta Z$ law and for reasonable choices about it, i.e. $1 < \Delta Y/\Delta Z < 5$ (the reader interested to recent discussions on this parameter and to the discrepancy between

theoretical and observational values is referred to Peimbert 1993 and Carigi et al. 1995), the He abundance that one has to adopt at $Z \leq 0.006$ is not very much different from the cosmological value $Y \approx 0.23$; the maximum variation is of about $+0.03$ at $Z=0.006$. For studying theoretically the influence of a variation of Y on the TRGB luminosity we have computed RGB models at $Z=0.0001$ and $Z=0.006$, adopting different initial He contents around $Y=0.23$. We find that $\frac{\partial M_{bol}^{tip}}{\partial Y}$ is ≈ 1.21 at $Z=0.0001$ and ≈ 0.75 at $Z=0.006$. Therefore, for reasonable choices of $\Delta Y/\Delta Z$, M_{bol}^{tip} will vary at most by only $\approx +0.02$ mag at $Z=0.006$.

Since very recently a new set of EOS tables, that constitutes an improvement in comparison with older tabulations, has been published (OPAL EOS, see Roger, Swenson & Iglesias 1996), we have also tested the influence of this new EOS on the theoretical determination of the TRGB luminosity. The OPAL EOS has been implemented in the evolutionary code as described in Paper II, and TRGB luminosities have been computed at the same metallicities previously quoted. The derived values of M_{bol}^{tip} are higher by not more than 0.02 mag in comparison with the ones derived from relation 1. Therefore we can consider relation 1 as representative of the theoretical canonical evolutionary determination of the TRGB luminosity (for metallicities $-2.35 \leq [M/H] \leq -0.57$, $Y=0.23$) adopting the most updated available input physics.

3 THE RGB TIP LUMINOSITY: COMPARISON WITH OTHER DISTANCE SCALES

For assessing the reliability of our TRGB luminosities for distance determinations, it is important to compare our results with observations of GCs and of nearby galaxies in which the stellar component has been resolved. In particular it is important to compare the distance moduli of the same object obtained by different distance indicators, verifying their mutual consistency. In the following we will separately discuss this kind of comparison for galactic GCs and for resolved galaxies.

3.1 Globular Clusters

In the case of galactic GCs it is possible to compare the distance scale fixed by ZAHB models, with the one derived from relation 1. Here we have adopted the same ZAHB distance scale presented in Paper II (the reader is referred to this paper for a comparison of our ZAHB models with observations), based on updated stellar models completely consistent with the RGB ones previously presented:

$$M_V^{zahb} = 1.129 + 0.388 \cdot [M/H] + 0.063 \cdot [M/H]^2 \quad (3)$$

in the same metallicity range as for equation 1, with a r.m.s.=0.011 mag. This relation provides the absolute visual magnitude M_V^{zahb} for the ZAHB at $\log T_{eff} = 3.85$ (that is approximately the average temperature of the RR Lyrae instability strip) as a function of the global amount of heavy elements $[M/H]$. By adopting the new OPAL EOS not only for RGB models but also for the ZAHB phase, luminosities brighter by ≈ 0.05 mag are obtained in comparison

with the values derived from relation 3 (see Paper II).

The comparison between the TRGB and the ZAHB distance scales fixed by equations 1 and 3 has been performed by adopting the TRGB observational data by Frogel, Persson & Cohen (1983 - hereinafter FPC83 - and reference therein), who provided absolute bolometric magnitudes for many TRGB of galactic GCs. These magnitudes have been obtained empirically by directly integrating the flux from the program stars via the observed *UBVJHK* photometry and adopting a RR Lyrae distance scale for the studied clusters. In this way we can compare directly the results from evolutionary computations with observations, without using transformations from the theoretical plane to the observational one.

Eleven clusters for which high resolution spectroscopic metallicity estimates and a good determination of their ZAHB level were available, have been selected from the FPC83 sample. The adopted ZAHB levels come from Paper II for 7 out of the 11 selected clusters (see paper II for the sources of the photometric data). For M71, NGC6352, NGC362 and M15 we have used the photometries respectively by Hodder et al. (1992), Fullton et al. (1995), Harris (1982) and Durrell & Harris (1993). In the case of M71, NGC6352 and NGC362, which have a red HB, the observational ZAHB level has been determined as described in Paper II for 47 Tuc, while for M15 we have adopted the same procedure described in Paper II for M68. For the clusters in common with DA90 we have adopted their reddening estimates, for M3 and M79 the values given by FPC83 have been used, while in the case of M71, NGC6352 and M68 we have used the values given by, respectively, Hodder et al. (1992), Fullton et al (1995) and Walker (1994).

In performing the comparison between RR Lyrae and TRGB distance scales we have used our RR Lyrae ZAHB distance scale (homogeneous with the TRGB one) determined by means of equation 3. The observational M_{bol}^{tip} values given by FPC83 were corrected for taking properly into account the differences with respect to our RR Lyrae distance scale and our adopted observational ZAHB luminosities. Moreover, following the prescriptions given by Frogel, Persson & Cohen (1978) the values of M_{bol}^{tip} have been also corrected (when necessary) for the difference between the reddenings adopted in this paper and the ones used by FPC83. The values of $[Fe/H]$ and $[\alpha/Fe]$ obtained by means of spectroscopic analysis, the global metallicity $[M/H]$ as derived from the spectroscopic determinations of $[Fe/H]$ and $[\alpha/Fe]$ and using the relation 2), reddening, the distance modulus (reddening corrected) and M_{bol}^{tip} are displayed in Table 2 for the sample of globular clusters we have selected.

In Figure 1 we have plotted the M_{bol}^{tip} values against the global heavy elements abundance for the selected clusters and also the theoretical relation for the TRGB luminosity (equation 1). The vertical error bar (± 0.1 mag) for the observational points represents an average error on the distance modulus obtained from relation 3 (see Paper II), while the error on the spectroscopic determination of $[M/H]$ is typically of about 0.15 dex (see Paper II).

In the case of M5 we have considered an error bar for the observational M_{bol}^{tip} equal to $-0.3, +0.1$ mag. This choice is consistent with the comments for this cluster in Table 29

Table 2. Data for the sample of selected galactic globular clusters.

Cluster	[Fe/H]	[α /Fe]	[M/H]	E(B-V)	$(m - M)_o$	M_{bol}^{tip}
M71	-0.80	0.27	-0.61	0.28	12.90	-3.56
NGC6352	-0.80	0.13	-0.70	0.21	13.82	-3.80
47 Tuc	-0.80	0.15	-0.70	0.04	13.19	-3.65
NGC362	-1.20	0.23	-1.04	0.06	14.50	-3.28
M5	-1.40	0.30	-1.19	0.03	14.30	-3.21
M79	-1.42	0.21	-1.27	0.00	15.62	-3.54
NGC6752	-1.50	0.31	-1.28	0.04	13.00	-3.44
M3	-1.49	0.26	-1.31	0.00	15.03	-3.37
NGC6397	-1.88	0.25	-1.70	0.18	11.83	-3.20
M68	-1.92	0.20	-1.78	0.07	14.87	-3.34
M15	-2.30	0.30	-2.09	0.10	15.03	-3.37

of FPC83. In fact FPC83 have emphasized the finding, in the central region of M5, of three stars that result to be up to 0.3 mag brighter than the brightest RGB star included in their analysis. This correction by 0.3 mag has been also adopted by DA90.

Let us mention again that the theoretical values for the TRGB luminosity have been converted to M_{bol} values by assuming the same value of the solar bolometric magnitude as adopted by FPC83 (i.e., $M_{bol,\odot} = 4.75$ mag).

From a first inspection of Figure 1 it is evident that the TRGB observational points are located at lower luminosities with respect to the theoretical relation, with an average difference of 0.20-0.25 mag. This means that TRGB and ZAHB distance scales agree at least at the level of ≈ 0.2 mag. However, it is worth bearing in mind that the observational determinations of GCs M_{bol}^{tip} as given by FPC83, provide only a lower limit to the ‘real’ maximum luminosity of the TRGB of GCs (see also the discussion in Castellani, Degl’Innocenti & Luridiana 1993 and Madore & Freedman 1995), since they have observed only a few stars in the upper part of the RGB of the studied clusters.

To go deeper into the investigation, we have studied if the observed distribution of the M_{bol}^{tip} values is compatible with the theoretical models, taking into account the statistical uncertainties due to the small sample of stars observed. For this aim, by using our evolutionary models, we have computed the time spent by a giant star in a given luminosity interval below the TRGB. The following relation has been obtained:

$$t \approx 9.67 \cdot (\Delta \log L) - 0.43 \cdot (\Delta \log L)^2 + 9.82 \cdot (\Delta \log L)^3 \quad (4)$$

where $\Delta \log L = \log(L/L_{\odot})^{tip} - \log(L/L_{\odot})$ and the time is in millions of years. This relation is largely independent of the metallicity.

From the ratio of evolutionary times in magnitude intervals $\Delta M_{bol} = 0.1$ mag, it is easy to estimate the expected distribution of \bar{N} stars along the last two bolometric magnitudes as given by $N_i = \bar{N} \cdot P_i$ where P_i is the probability to find one single star in the chosen interval (and it is equal to the ratio between the time spent in the selected M_{bol} interval and the total time spent in the two last bolometric magnitudes below the TRGB). Adopting from FPC83 $\bar{N} = 20$ as a typical value for clusters in our sample and using a binomial distribution, the probability to find at least one star in a chosen luminosity interval below the tip is equal to:

Figure 1. The bolometric magnitude of the brightest observed red giant as a function of the global metallicity, for the sample of clusters selected from the FPC83 database. The solid line shows the theoretical expectation for the bolometric magnitude of the RGB tip for an age of 15Gyrs. The dashed lines represent the same theoretical relation but shifted at step of 0.1 mag (see text).

$$P_1 = \sum_{n=1}^{20} \frac{20!}{n! \cdot (20-n)!} \cdot P_i^n \cdot (1-P_i)^{20-n} \quad (5)$$

By using this relation and equation (4), one obtains that it exists a probability of 56% of finding at least one star within 0.2 mag below the TRGB, 71% within 0.3 mag and, 81% within 0.4 mag below the tip. These values are in agreement with the data shown in figure 1. Therefore, in the case of galactic GCs, the TRGB and ZAHB distance scales given by relations 1 and 3 agree within the statistical uncertainty due to the small stellar sample considered in the observational data. It is worth noting that if we had used the ZAHB and RGB models computed by adopting the OPAL EOS, the agreement between the two distance scales would be not degraded, since the variations are small, especially if compared with the typical observational error. In fact the points in figure 1 would be located at luminosities higher by ≈ 0.05 mag (due to the more luminous ZAHB and therefore higher distance moduli) and the solid line would be lowered by only ≈ 0.01 mag (due to the slightly lower TRGB luminosity).

DA90 performed a similar comparison for a sample of 8 GCs, but they used the HB distance scale by Lee, Demarque & Zinn (1990) and the theoretical RGB models by Sweigart & Gross (1978), computed with old input physics. Their results were different from ours, since they obtained that the

Table 3. Dereddened (V-I) color of the RGB - taken at $M_I = -3.5$ mag - and metallicity for a sample of globular clusters. The distance moduli of these clusters have been obtained adopting our ZAHB distance scale and are displayed in Table 2.

Cluster	[M/H]	$(V - I)_{0,-3.5}$
M15	-2.09	1.235
NGC6397	-1.70	1.272
NGC6752	-1.28	1.401
M5	-1.19	1.450
NGC362	-1.04	1.489
47Tuc	-0.70	1.860

observed values of M_{bol}^{tip} were brighter than the theoretical prescription by ≈ 0.10 mag. Moreover, the statistical correction due to the small sample of RGB stars considered would increase this discrepancy. On the contrary, we find that the observational values of M_{bol}^{tip} are less bright than the theoretical counterpart by an amount that is in agreement with the statistical expectation previously described. This is an important confirmation of the consistency between RGB and HB evolutionary models, and of the reliability of the input physics used in computing the stellar models. In particular, it can be also deduced that there cannot be a significant ‘missing physics’ that could modify appreciably the degenerate He core masses at the TRGB, which fix the TRGB and the ZAHB luminosities.

3.2 Resolved galaxies

Another test for checking the consistency of the TRGB luminosity with independent distance indicators can be performed by considering resolved galaxies, for which the distance modulus can be derived by using at the same time TRGB, RR Lyrae stars and Cepheids observations.

LMF93 suggested an iterative procedure for determining the distance of a galaxy from observations in the VI Johnson-Cousins bands, by adopting the TRGB as distance indicator. Such procedure can be summarized as follows (see LFM93 for more details):

- i) fixing preliminarily the distance modulus;
- ii) with the fixed distance modulus determining the metallicity by measuring the dereddened color at $M_I = -3.5$ mag and using the relation: $[Fe/H] = -12.64 + 12.6 \cdot [(V - I)_{0,-3.5}] - 3.3 \cdot [(V - I)_{0,-3.5}]^2$;
- iii) obtaining the distance modulus from the observed I magnitude of the TRGB (corrected for the interstellar extinction) by adopting empirical relations (from DA90) for both the TRGB bolometric magnitude as a function of metallicity and the bolometric correction to the I magnitude;
- iv) iterating the previous steps until convergency is obtained between the distance modulus at step (i) and the one obtained after step (iii).

Therefore in order to estimate the distance modulus of a galaxy by adopting the TRGB method, from a theoretical point of view, one needs relations providing the bolometric correction in the I band and the bolometric magnitude of the TRGB. As far as the bolometric magnitude of the TRGB is concerned, we have already provided a relation which gives the value of M_{bol}^{tip} as a function of the global amount of heavy elements (equation 1). Following LFM93,

an empirical $BC_I - (V - I)_0$ relation for RGB stars has been taken from DA90. In that paper the authors give:

$$BC_I = 0.881 - 0.243 \cdot (V - I)_0 \quad (6)$$

independent of the metallicity, with a dispersion of 0.057 mag (see their Fig. 14). In deriving this relation DA90 have considered VI photometry (from their observations and from Lloyd Evans 1983) and the corresponding M_{bol} values (from FPC83) for stars along the RGBs of 47Tuc, NGC362, NGC1851, M5, NGC6752, ω Cen, NGC6397 and M15, spanning a wide range of $(V - I)$ and $[M/H]$ values. The colors of the stars have been corrected for the reddening of the clusters adopting the $E(V - I)/E(B - V)$ relation of Dean, Warren & Cousins (1978), while the I magnitudes and the M_{bol} values have been adjusted to the theoretical RR Lyrae distance scale given by Lee, Demarque & Zinn (1990). The difference between M_I and M_{bol} gives the requested bolometric correction to the I magnitude. It is important to note that, as it is evident, once the I and M_{bol} data are adjusted to the same distance scale, the derived BC_I is independent on which distance scale is assumed.

Since the bolometric magnitude of the TRGB depends on the metallicity, and for very distant objects it is not possible to perform accurate high resolution spectroscopical determinations of $[M/H]$ as in the case of galactic GCs, it is important to obtain a relation providing the heavy elements abundance as a function of some observable quantity. It is well known that the RGB location in the HR diagram ranks with the metallicity in the sense that the RGB becomes redder and less steep with increasing $[M/H]$ value. Therefore one can relate the dereddened color of the RGB at a fixed absolute magnitude, for instance at $M_I = -3.5$ mag ($(V - I)_{0,-3.5}$), following the choice made by LFM93, with the global amount of heavy elements. We have selected from the database of DA90, a sample of clusters for which high resolution, spectroscopical determinations of $[M/H]$ are available (see Paper I) and have performed a cubic regression of the cluster metallicity versus $(V - I)_{0,-3.5}$ (see data in Table 3 and figure 2), imposing that the $(V - I)_{0,-3.5}$ values have to be monotonously increasing for increasing metallicity, obtaining:

$$[M/H] = -45.16 + 73.71 \cdot [(V - I)_{0,-3.5}] - 40.91 \cdot [(V - I)_{0,-3.5}]^2 + 7.59 \cdot [(V - I)_{0,-3.5}]^3 \quad (7)$$

with a dispersion of 0.04 dex. The errors displayed in figure 2 correspond to 0.15 dex in $[M/H]$ and to about 0.04 mag in $(V - I)_{0,-3.5}$, that is the average variation of $(V - I)_{0,-3.5}$ due to an uncertainty of about 0.10 mag in the adopted clusters distance moduli.

By adopting relations 1, 6 and 7 it is possible to apply the TRGB method for distance determinations - as described by LFM93 - to nearby resolved galaxies, according to the relation:

$$(m - M)_I = I_{TRGB} + BC_I - M_{bol}^{tip} \quad (8)$$

An alternative approach for applying the TRGB method to resolved galaxies is to adopt theoretical BC_I values derived from model atmospheres. Courtesy of Dr. F. Castelli, we

Figure 2. Metallicity calibration: $[M/H]$ values from Paper I, together with their associated uncertainties, are plotted against $(V - I)_{0,-3.5}$. The solid line shows a cubic fit to these points (see text).

have been able to use new color transformations, obtained with an updated version of the Kurucz's code ATLAS9, which constitute a significant improvement in comparison with old available evaluations (Castelli 1996, private communication). By adopting these new transformations, we have transformed our tracks from the theoretical plane into the observational one and we have analyzed the behavior of the I (Cousins) magnitude of the TRGB *versus* the metallicity. The following relation has been obtained, performing a best fit to the data:

$$M_I^{tip} = -3.732 + 0.588 \cdot [M/H] + 0.193 \cdot [M/H]^2 \quad (9)$$

with a r.m.s.=0.008, spanning the same metallicity range as equation 1.

We have therefore applied the TRGB method by adopting both the empirical bolometric corrections given by equation 6, or the results from theoretical model atmospheres (equation 9). In this way we have checked the consistency with the distances obtained independently by adopting the Cepheid and our RR Lyrae (equation 3) distance scales and, we have also tested the agreement between empirical and theoretical BC_I .

The observational database is the same one collected by LFM93, with the exception of the LMC RR Lyrae distances (see discussion below) and with the additional data for Sextans A taken from Sakai et al. (1996). The A_I extinction has been treated as in LFM93 (see also Madore & Freedman 1991, Lee 1993).

Figure 3. Comparison of distances for the selected sample of resolved galaxies, obtained using the TRGB (both empirical and theoretical BC_I values) and the Cepheid distance scale.

As far it concerns the Cepheid distance scale, recently a big effort has been made for accounting for metallicity effects in the pulsational properties of Cepheid stars (Bono 1996, private communication) and for metallicity and/or interstellar reddening effects in the observational calibrations of the Cepheid distance scale (see Laney & Stobie 1995, Di Benedetto 1994 and references therein). However, an analysis of the problems and of the recent theoretical and observational improvements in the calibration of the cosmic distance scale through Cepheid variable stars is beyond the scope of the present paper; here we have adopted the same distance scale used by LFM93.

In Table 4 we report the distance modulus determinations as obtained with the three different methods. The various columns provide the following data: (1) the name of the object; (2) the morphological type (as given by Sandage & Tammann 1987); (3) the reddening; (4) the observed I magnitude of the TRGB; (5) the mean RGB metallicity, as obtained adopting relation 7; (6) the distance modulus estimated by using the TRGB method; (7) the intrinsic Cepheid distance; (8) the distance obtained by using the RR Lyrae luminosity; (9) as in column (8) but for an average metallicity of the RR Lyrae population $[M/H]=-1.5$ (see below); (10) the distance modulus obtained applying equation 9.

The TRGB luminosities determined for this sample of galaxies are based on observations of a large number of RGB stars. According to Madore & Freedman (1995), for avoiding an underestimation of the I TRGB magnitude due to population effects (as discussed previously) a sample of about 50 - 100 stars in the upper magnitude interval has to be observed. In the case of these resolved galaxies the RGB star

Table 4. Selected parameters (see text) for a sample of resolved galaxies.

Galaxy	Type	$E(B - V)$	I_{TRGB}	$[M/H]$	$(m - M)_{TRGB}$	$(m - M)_{Ceph}$	$(m - M)_{RR}$	$(m - M)_{RR}^{-1.5}$	$(m - M)_{TRGB}^{theor}$
LMC	SBmII	0.10	14.60	-1.06	18.59	18.50	18.40		18.53
NGC6822	ImIV-V	0.28	20.05	-1.75	23.61	23.62			23.64
NGC185	dE3pec	0.19	20.30	-1.06	24.11		23.93	24.01	24.05
NGC147	dE5	0.17	20.40	-0.85	24.28		24.00	24.13	24.14
IC1613	ImV	0.02	20.25	-1.15	24.44	24.42	24.21	24.27	24.36
M31	SbI-II	0.08	20.55	-0.81	24.58	24.44	24.31	24.45	24.47
M33	Sc(s)II-III	0.10	20.95	-2.05	24.82	24.63	24.69	24.62	24.87
WLM	ImIV-V	0.02	20.85	-1.48	24.97	24.92			24.99
NGC205	S0/dE5pec	0.035	20.45	-0.81	24.56		24.68	24.82	24.46
Sex A	ImV	0.075	21.79	-1.80	25.91	25.85			25.80
NGC3109	SmIV	0.04	21.55	-1.48	25.61	25.50			25.65

metallicities. In the case of the LMC we have considered the average RR Lyrae luminosities for 5 clusters (NGC1466, NGC1835, NGC1841, NGC2257, Reticulum) with a number of observed RR Lyrae stars larger than 20. The observational data, the reddenings and the $[Fe/H]$ values for these clusters are taken from Walker (1992). We have derived the clusters distance moduli assuming $[\alpha/Fe]=0.0$ or 0.3, two different corrections due to the LMC geometry (as described by Walker 1992) and no correction. After averaging the five distance moduli for each of the six cases considered, we have obtained a LMC distance modulus ranging between 18.38 and 18.44; in Table 4 we have reported the value derived without corrections for the LMC geometry, and assuming $[\alpha/Fe]=0$.

The errors associated to the RR Lyrae distance scale derived via equation 3 are probably higher than in the case of the TRGB, at least when the observations adopted were performed in the g Thuan-Gunn band (as in the case of NGC185, NGC147, IC1613, NGC205). In fact, in this case it is necessary to transform the $\langle g \rangle$ values for the mean RR Lyrae luminosity to $\langle V \rangle$, before applying the correction for the evolution off the ZAHB and equation 3. But after the pioneristic work of Thuan & Gunn (1976), such transformation has not been studied in details, so up to date its associated uncertainty is unknown. For such a reason, it could be possible that the use of this transformation law (in particular we have used the prescription given by Saha & Hoessel 1990) introduces an additional uncertainty to the final derived distance modulus. Moreover, it is worth bearing in mind that, even if Saha & Hoessel (1990) have shown that the presence of non RR Lyrae pulsators does not significantly affect the estimate of the mean magnitude obtained from the peak of the magnitude distribution of *all* pulsating stars, it is possible that in some stellar system such occurrence could introduce a not negligible uncertainty in the *real* mean RR Lyrae magnitude.

Another important source of uncertainty for the RR Lyrae distance modulus is related to the metallicity of the RR Lyrae population. The $[M/H]$ value used for obtaining the distance moduli given in column 8 of Table 4 are derived from RGB stars, and represent an average metallicity of this stellar population. In principle this metallicity could not correspond to the RR Lyrae average metal content, as for instance for the two “metal-rich RGB” galaxies M31 and NGC205, due to the low probability that metal-rich RGB stars evolve during their He central burning phase through the RR Lyrae instability strip. For roughly estimat-

Figure 4. Comparison of distances for the selected sample of resolved galaxies, obtained using the TRGB (by means of equation 9), the Cepheid and the RR Lyrae (equation 2) distance scales. The comparison between the distance moduli obtained using the TRGB method and the RR Lyrae distance scale but adopting for the RR Lyrae stellar population an average metallicity equal to $[M/H]=-1.5$ (see text) is also shown.

sample is large enough to satisfy this requisite. Therefore we can in principle compare directly observed and predicted TRGB I luminosities, without taking into account the statistical uncertainty due to the size of the star sample. The typical observational errors on the estimated distance moduli with the TRGB or the Cepheids are of the order of ≈ 0.15 mag (see, i.e. LFM93, Sakai et al. 1996).

The RR Lyrae distance moduli displayed in column 8 of Table 4 are obtained considering the observed RR Lyrae average magnitude, applying the correction for the off ZAHB evolution given by equation 5 of Paper II, and then using equation 3; the metallicities considered in this procedure are given in column 5 and correspond to the mean RGB

ing the additional uncertainty introduced by the unknown metallicity of the RR Lyrae population, the distance moduli obtained assuming for the RR Lyrae stars an average metallicity equal to $[M/H]=-1.5$ - adopted as a reasonable estimation of the average metallicity for the galactic GCs RR Lyrae population - have been also reported (with the unique exception of the LMC) in Table 4 (column 9).

In figure 3, we display the difference between the distance moduli obtained by adopting the TRGB and the Cepheid distance scale with respect to the TRGB distances (by using both empirical and theoretical BC_I values). As it is evident from the figure, the agreement between these three different distance scales is remarkably satisfactory. In particular, the average difference between the TRGB distance moduli obtained by adopting empirical BC_I values and the Cepheid distances is equal to only $\approx +0.08$ mag, while in the case of theoretical BC_I values the same average difference is $\approx +0.05$ mag.

In Figure 4, the differences between TRGB distance scale fixed by equation 9 or the Cepheid distance scale with respect to the RR Lyrae ones (columns 8 and 9 of Table 4) are displayed as a function of the RR Lyrae distance moduli. When considering for the population of RR Lyrae stars metallicities equal to the $[M/H]$ values obtained from the RGB color (column 5 in Table 4), the average difference between RR Lyrae and Cepheid distance moduli is of about 0.14 mag (the RR Lyrae distance moduli being lower), while between RR Lyrae and TRGB is of about 0.12 mag (again the RR Lyrae distance moduli being lower). If we don't consider NGC205, the average difference between RR Lyrae and TRGB distance moduli for the remaining six galaxies results to be of about 0.18 mag, with a much smaller scatter, the RR Lyrae distance moduli being systematically lower. By adopting the OPAL EOS in the evolutionary computations, the distance moduli obtained using the RR Lyrae distance scale should be increased by about 0.05 mag (whereas, as discussed in section 2 the TRGB luminosity is almost unaffected by adopting the OPAL EOS in the computations). In this case the difference between the RR Lyrae and the Cepheid distance scales is reduced to ≈ 0.10 mag, while the difference with respect to the TRGB distance scale is of about 0.13 mag (not considering NGC205).

If we consider the distance moduli obtained assuming $[M/H]=-1.5$ for RR Lyrae stellar populations (and the LMC distance modulus given in column 8 of Table 4) the average difference between RR Lyrae and TRGB distance moduli (still neglecting NGC205) is equal to ≈ 0.09 mag (0.08 mag if for M33 we consider its metal poor RGB metallicity), reduced to ≈ 0.04 mag when taking into account the effect of OPAL EOS on the ZAHB models.

In particular, when comparing the LMC RR Lyrae distance modulus (that probably is more accurate than for the other galaxies and it is less affected from the uncertainties on the real RR Lyrae metallicity) with the TRGB one, we find a difference of 0.13 mag (reduced to ≈ 0.08 mag if the OPAL EOS is used in the stellar models), well within the errors that are typically of the order of 0.15 mag for the TRGB and at least of 0.10 mag for the RR Lyrae distances.

Therefore from the data presented in this section and in section 3.1 it comes out that, within the observational uncertainties and in the limit of the small sample of GCs and resolved galaxies considered, it does not appear to exist a

clear inconsistency between ZAHB and RGB stellar models; the TRGB and RR Lyrae distance scales set by the evolutionary calculations agree at the level of ≈ 0.1 mag, while the TRGB and Cepheid distance scales are consistent within less than 0.1 mag.

4 SUMMARY AND CONCLUSIONS

We have performed a study about the self-consistency of theoretical RGB and ZAHB stellar models, computed with the most updated input physics. For this aim and for providing an updated distance scale based on the TRGB method, new theoretical M_{bol}^{tip} - $[M/H]$ and M_I^{tip} - $[M/H]$ relations have been provided together with a recalibration of the empirical $[M/H]-(V-I)_{0,-3.5}$ relation (based on recent high resolution metallicity determinations for galactic GCs).

The comparison between the TRGB distance scale and the RR Lyrae one, adopting galactic GCs observations, has disclosed a good agreement within the statistical uncertainty due to the size of the observed clusters stellar sample presently available.

We have presented also a comparison among TRGB, RR Lyrae and Cepheid distance scales, employing recent photometric studies of nearby resolved galaxies. The following results have been obtained:

- i) the distance moduli determined by adopting our M_I^{tip} - $[M/H]$ relation, or M_{bol}^{tip} - $[M/H]$ together with the empirical BC_I values by DA90, or the Cepheid distance scale by LFM93, agree on average within less than 0.1 mag; in particular, it is remarkably good the consistency between the completely theoretical prescription for the I_{TRGB} luminosities given by equation 9 and the Cepheid distance scale (distance moduli different on average by only 0.05 mag).
- ii) the agreement between RR Lyrae and TRGB (or Cepheid) distance scales when considering the sample of resolved galaxies is at the level of ≈ 0.1 mag, taking into account the uncertainties on the metallicity of the RR Lyrae populations. The data from GCs are consistent with no discrepancy between TRGB and RR Lyrae distances;
- iii) the $[M/H]$ values obtained by adopting the recalibrated $[M/H]-(V-I)_{0,-3.5}$ relation are higher by not more than 0.15 dex in comparison with the $[Fe/H]$ values derived by LFM93;
- iv) the distance moduli derived adopting our theoretical M_I^{tip} - $[M/H]$ relation, together with our estimations of the global metallicities are on average ≈ 0.11 mag higher than the values obtained by LFM93.

The satisfactory agreement between the three discussed distance scales allows to assess the reliability of the presented theoretical TRGB and ZAHB luminosities. In particular, the provided theoretical relations for the TRGB luminosity (equations 1 and 9) can be safely used when adopting the TRGB as a distance indicator for resolved galaxies, but also for GCs, if a sufficiently populated RGB can be observed.

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